Lesson 4

THE MEANING OF DIVISION

In this Lesson, we will address the following:

1. **WHAT IS THE PROBLEM IN DIVISION?**
2. **HOW DO WE DIVIDE A NUMBER INTO EQUAL PARTS?**

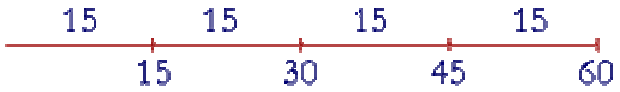
IN MULTIPLICATION we are given two numbers and we have to find their product.

$4 \times 15 = ?$ $4 \times 15 = 60.$

But when we know the product, and we ask "What number *times* 15 equals 60?" $? \times 15 = 60$

that is called the inverse of multiplication. To answer, we have to "divide." $60 \div 15 = ?$

We have to *divide* 60 into groups of 15:



We can divide 60 into *four* 15's. $60 \div 15 = 4$ "60 divided by 15 equals 4" (or is equal to 4)

Equivalently, we could *subtract* 15 from 60 four times. Multiplication is repeated addition. Therefore we can think of division as repeated subtraction.

15 is called the divisor, that is how 60 will be divided. 60 is called the dividend; it is the number being divided. 4 is called the quotient.

1. WHAT IS THE PROBLEM IN DIVISION?

$60 \div 15 = ?$ or $? \times 15 = 60$

Given two numbers, called the *dividend* and the *divisor*, we are to find the number of times the divisor is *contained* in the dividend.
Equivalently, we are to find what number *times* the divisor will equal the dividend.
We can also think of it as finding the number of times we could *subtract* the divisor from the dividend.
That number of times is called the *quotient*.

Example 1. What number times 6 equals 18?

Obviously, 3 × 6 = 18. On writing this with the division sign ÷, we get 18 ÷ 6 = 3

"18 divided by 6 equals 3."

6 is the number to the right of the division sign ÷; it is the divisor. 18 is the dividend. 3 is the quotient.

That is,

6 × 3 = 18 Dividend ÷ Divisor = Quotient

Quotient × Divisor = Dividend

Note: A divisor may not be 0, like 6 ÷ 0, because any number *times* 0 will still be 0. Therefore division by 0 is an excluded operation.

As for 0 ÷ 0, that could be *any* number, because *any* number times 0 equals 0!

Example 2. What number times 10 will equal 72?

Answer. Any question that asks "What number times. . .?" is a division problem. We have to divide 72 by 10. On separating one decimal place:

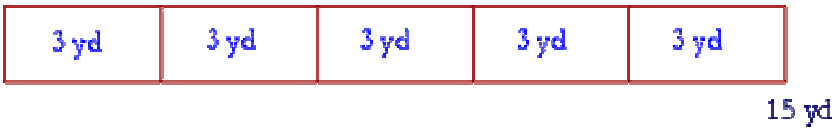
72 ÷ 10 = 7.2

That is,

7.2 × 10 = 72

Example 3. How many pieces of cloth 3 yards long could you cut from a piece that is 15 yards long?

Answer. You could cut 5 pieces, because 5 × 3 yd = 15 yd.



15 yd ÷ 3 yd = 5.

15 yards have been *divided* - broken up - into five lengths of 3 yards each. 3 goes into 15 five times. We could subtract 3 from 15 five times.

This problem illustrates the following point: The dividend and divisor must be units of the same kind. We can only divide - repeatedly subtract - yards from yards, dollars from dollars, hours from hours. We cannot divide 8 apples by 2 oranges.

8 apples ÷ 2 oranges = ? because no number *times* 2 oranges will equal 8 apples!

Example 4. A bus is scheduled to arrive every 12 minutes. In the course of 2 hours, how many buses will arrive?

Solution. How many times is 12 minutes contained in 2 hours? But the units must be the same. Since 1 hour = 60 minutes, then 2 hours = 2 × 60 = 120 minutes.

Therefore,

120 minutes ÷ 12 minutes = 10, because 10 *times* 12 minutes = 120 minutes.

In the course of 2 hours, 10 buses will arrive.

2. WHICH NUMBERS WILL BE EXACTLY DIVISIBLE BY A GIVEN NUMBER?

Answer: The *multiples* of that number.
The numbers exactly divisible by 3 are the multiples of 3:

3, 6, 9, 12, and so on.

And since these are divisible by 3, so are

30, 60, 90, 120, . . .

300, 600, 900, 1200, . . .

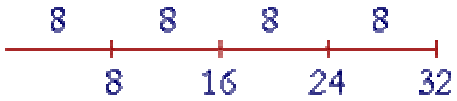
The numbers exactly divisible by 8 are the multiples of 8:

8, 16, 24, 32, . . .

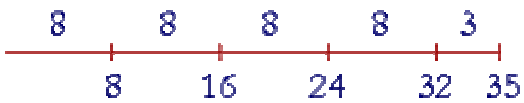
80, 160, 240, 320, . . .

800, 1600, 2400, 3200, . . .

The figure below illustrates that 8 goes into 32 exactly. 32 is a multiple of 8.



But 8 does not go into 35 exactly:



35 is not a multiple of 8. There is a remainder of 3 35 ÷ 8 = 4 R 3

The remainder is what we have to *add* to the multiple of 8 to get 35 $35 = 4 \times 8 + 3$

Possible remainders: say that there is a large group of people, and we want to divide them into

groups of 5.

But say we discover that there is not an exact multiple of 5. Then how many people might we *not* be able to group? How many people might remain?

Answer: Either 1, or 2, or 3, or 4. Because if more than 4 remained, we could make another group of 5!

The point is:

The remainder is always less than the divisor.

If we divide by 5, then the possible remainders are 1, 2, 3, or 4.

Example 5.

a) If 7 is the divisor, what are the possible remainders? **Answer.** 1, 2, 3, 4, 5, 6.

b) How many 7's are there in 61? **Answer.** 8. And there is a remainder of 5 $61 \div 7 = 8 \text{ R } 5$

That is, $61 = 8 \times 7 + 5$

Example 6. Prove: $47 \div 9 = 5 \text{ R } 2$

Proof. $47 = 5 \times 9 + 2$.

Example 7.

Divide 53 by 8. Write the whole number quotient and the remainder. **Answer**. $53 \div 8 = 6 \text{ R } 5$

8 goes into 53 six (6) times: 48. What must we add to 48 to get 53? 5.

$48 + 5 = 53$ 5 is the remainder.

The division bar: in what follows, we will signify division in this way:

$$\frac{16}{8} = 2$$

"16 divided by 8 is 2."

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient}$$

The test is:

Quotient × Divisor = Dividend

The short horizontal line is called the division bar. The division bar is also used to signify a fraction, because a fraction indicates division of the numerator by the denominator. We also use the division bar to signify the ratio of two numbers.

Example 8. $\frac{280}{7} = ?$

"280 divided by 7 is what number?"

Answer. Ignore the 0. 7 goes into 28 four (4) times. Therefore 7 goes into 280 forty (40) times.

$$\frac{280}{7} = 40.$$

40 *times* 7 is 280.

In other words, since 28 is divisible by 7, then so is '28' followed by any number of 0's.

280 2800 28,000 280,000 . . .

Example 9. $\frac{5400}{9} = ?$

Answer. 600. Because 600 × 9 = 5400.

Example 10. $\frac{246}{100} = 2.46$

This is 246 ÷ 100.

Example 11. $5 \div 8 = \frac{5}{8}$

$5 \div 8$ is simply the fraction $\frac{5}{8}$, which we can interpret as $5 \div 8$

because of the division bar