

Lesson 8

RATIO AND PROPORTION (second half)

In this Lesson, we will answer the following:

3. HOW CAN WE EXPRESS THE RATIO OF ANY SMALLER NUMBER TO A LARGER?
4. WHEN IS A RATIO EXPRESSED WITH THE LOWEST TERMS?
5. HOW CAN WE KNOW WHEN THE RATIO OF TWO NUMBERS IS EXPRESSED WITH THE LOWEST TERMS?
6. WHAT IS THE THEOREM OF THE COMMON DIVISOR?

3. How can we express the ratio of any smaller number to a larger as 2 to 3?

Let each number say its name. Let the smaller number say its cardinal name: One, two, three,.... Let the larger number say its ordinal name: third, fourth, fifth,....

Example 1. What ratio has 2 to 3?

Answer. "2 is two thirds of 3."

2 says its cardinal name, "Two." 3 says its ordinal name, "third."

That is how to express the ratio of any smaller number to a larger.

Example 2. What ratio has 4 to 5?

Answer. "4 is four fifths of 5."

To see this, consider that 1 is one fifth of 5:



2 is two fifths of 5.

3 is three fifths of 5.

4 is four fifths of 5.

Each number says its name.

4. When is a ratio expressed with the lowest terms?

When it is expressed with the smallest numbers which have that ratio.

Many pairs of numbers have the same ratio as 4 and 12, for example. 5 and 15, 6 and 18, 10 and 30. And so on. But the smallest pair are 1 and 3.

4 is to 12 as 1 is to 3.

Now, it is perfectly correct to say "4 is four twelfths of 12".

But when we say "4 is one third of 12",

then we have expressed that ratio with the *lowest* terms -- with the smallest numbers which have that ratio. The terms 1 and 3 say their names.

5. How can we know when the ratio of two numbers is expressed with the lowest terms?

1 is their only common divisor.

Example 3. The ratio of 7 to 8 expressed as seven eighths is with the lowest terms, because 1 is the only common divisor of 7 and 8. But the ratio of 12 to 18 expressed as twelve eighteenths, although true, is not with the lowest terms, because 12 and 18 have a common divisor 6.

6. What is the Theorem of the Common Divisor?

"If two numbers are divided by the same number, then the quotients will have the same ratio as the numbers divided."

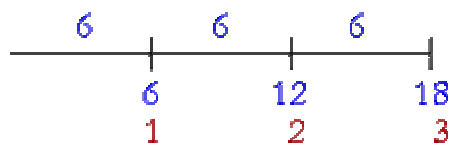
Example 4. What ratio has 12 to 18?

Answer. 12 and 18 have a common divisor 6:

"6 goes into 12 two times and into 18 three times."

The quotients are 2 and 3. Therefore according to the theorem, 12 is to 18 as 2 is to 3.

"12 is two thirds of 18."



2 and 3 say their names.

Example 5. Express the ratio of 21 to 28 with the lowest terms.

Answer. Their ratio will be expressed with a sentence,

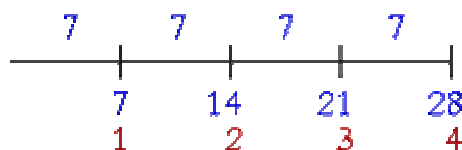
"21 is _____ 28."

21 and 28 have a common divisor 7:

"7 goes into 21 three times and into 28 four times."

21 is to 28 as 3 is to 4.

"21 is three fourths of 28."



Notice that their common divisor, 7, is one fourth of 28.

Example 6. If a is an eighth of b , then a is to b as ? is to ?.

Complete that proportion with the smallest numbers -- the lowest terms -- which have that ratio.

Answer. There are many pairs such that the first is an eighth of the second. 2 and 16; 5 and 40; 10 and 80. However, the smallest such numbers are 1 and 8.

If a is an eighth of b , then a is to b as 1 is to 8.

Example 7. Numbers that end in the same number of 0's. What ratio has 800 to 2400?

Answer. *If two numbers end in the same number of 0's, then we may ignore those 0's and take the ratio of what remains.*

Both 800 and 2400 end in two 0's. Therefore 800 is to 2400 as 8 is to 24.

8 is a third of 24. Therefore, 800 is a third of 2400.

Why may we ignore the 0's? Because when we do, we have divided both terms by the same power of 10.

The quotients will have the same ratio as the numbers divided.

Example 8. What ratio has 400 to 50?

Answer. Since 400 is larger than 50, it will be so many times 50.

Now, 50 ends in only one 0, therefore we may ignore only one 0:

400 is to 50 as 40 is to 5.

40 is eight times 5. Therefore, 400 is eight times 50.