

Lesson 7

RATIO AND PROPORTION (first half)

In this Lesson, we will answer the following:

1. WHAT DO WE MEAN BY THE RATIO OF TWO NATURAL NUMBERS?
2. WHAT IS A PROPORTION?

1. What do we mean by the ratio of two natural numbers?

It is their *relationship* with respect to relative size which we can express in words. Specifically, one number is a *multiple* of the other (so many times it), a part of it, or *parts* of it.

Example 1. Multiple. What ratio has 15 to 5?

Answer. 15 is three times 5.

That is the ratio -- the relationship -- of 15 to 5. If Jill has \$15, and Jack has \$5, then Jill has three times more than Jack.

We do not answer "3 to 1," because we want to name the ratio of 15 to 5 explicitly. It is true that 15 is to 5 as 3 is to 1 -- but what ratio has 3 to 1? 3 is three times 1.

(The 19th century program to rid mathematics of language and replace it with algebraic relations, successfully put to sleep the subject of ratio and proportion.)

Notice that we answer with a complete sentence beginning with the first number 15 and ending with the last number 5. For, a ratio is a relationship.

The two numbers in a ratio are called the terms; the first and the second.

When the first term is larger, we say it is so many *times* the smaller number. 15 is three times 5.

What ratio has 28 to 7?

28 is four times 7.

Example 2. Part. What ratio has 5 to 15?

Answer. 5 is the third part of 15.

That is called the inverse ratio of 15 to 5. The terms are exchanged.

Notice again that we answer with a complete sentence beginning with the first term and ending with the second. "5 is _____ 15."

Example 3. Parts. What ratio has 10 to 15?

Third Third Third
5 10 15

Answer. 10 is two thirds of 15.

These are the three types of ratio. One number is a multiple of the other (so many times it), a part of it, or parts of it.

Example 4. What ratio has 12 to 6?

Answer. 12 is two times 6. Or we could say, "12 is twice as much as 6," or "12 is double 6."

These are the various ways of expressing the ratio, the relationship, of 12 to 6.

Inversely, 6 is half of 12.

When trying to express a ratio, if the student will say a sentence, and then consider the truth of that sentence, the fact will speak for itself.

Example 5. What ratio has 6 to 30? And inversely, what ratio has 30 to 6?

Answer. 6 is the fifth part of 30. Inversely, 30 is five times 6.

Example 6. What ratio has 63 to 7? And inversely, what ratio has 7 to 63?

Answer. 63 is nine times 7. Inversely, 7 is the ninth part of 63.

2. What is a *proportion*?

A statement that two ratios are the same.

Example 1. 5 is to 15 as 8 is to 24.

This is a proportion because 5 is the **third part** of 15, just as 8 is the **third part** of 24. A proportion involves four numbers -- four **terms**. To explain *why* they are proportional we start with the first term and state its ratio to the second; then we state the ratio of the third term to the fourth.

Example 2. Why is this a proportion?

16 is to 2 as 80 is to 10.

Answer. This is a proportion because 16 is **eight times** 2, just as 80 is **eight times** 10.

Example 3. Why is this a proportion?

10 is to 15 as 2 is to 3.

<u>Third</u>	<u>Third</u>	<u>Third</u>
5	10	15
1	2	3

Answer. This is a proportion because 10 is **two thirds** of 15, just as 2 is **two thirds** of 3.

Example 4. Complete this proportion:

8 is to 32 as 9 is to ?

Solution. 8 is the **fourth part** of 32. And 9 is the **fourth part** of 36.

The 1st term is to the 2nd as the 3rd is to the 4th.

Example 5. Complete this proportion:

27 is to 3 as ? is to 5

Solution. 27 is **nine times** 3. And 45 is **nine times** 5.

If the student will speak and use sentences, the answer will be clear.

Example 6. In each item below, what ratio has a to b ?
(a simply means the first term; b means the second.)

a) a is to b as 1 is to 6.

b) a is to b as 10 is to 1.

Answers.

a) Since 1 is the **sixth part** of 6, then a is the **sixth part** of b .

b) Since 10 is **ten times** 1, then a is **ten times** b .

Symbolic representation

We will let the following signify a proportion:

$$\frac{1}{2} = \frac{4}{8}$$

Read this: "1 is to 2 as 4 is to 8."

Why is this a proportion? Because 1 is half of 2, and 4 is half of 8.
(Can you be innocent and make-believe you never heard of a "fraction"? Fractions, as we will see, are based on ratios, not vice-versa.)

Example 7. Read this proportion, and complete it:

$$\frac{8}{2} = \frac{20}{?}$$

Answer. "8 is to 2 as 20 is to what number?"

Now, what ratio has 8 to 2?

8 is **four times** 2. And 20 is **four times** 5.

What is taught in most textbooks these days as "ratio and proportion," is not. The innocent student is taught to write the letter x for the unknown term, cross-multiply, and solve an algebraic equation. While that mechanical method gives the answer, it offers no understanding of why. It is not, therefore, educational.

A ratio is relationship expressed in words. It requires understanding.

Example 8. Complete this proportion:

$$\frac{7}{21} = \frac{4}{?}$$

Answer. "7 is to 21 as 4 is to what number?"

What ratio has 7 to 21?

7 is the **third part** of 21. And 4 is the **third part** of 12.

Example 9. Complete this proportion:

$$\frac{2}{3} = \frac{?}{12}$$

Answer. "2 is to 3 as what number is to 12?"

Now, 2 is **two thirds** of 3. What number is **two thirds** of 12?

<u>Third</u>	<u>Third</u>	<u>Third</u>
1	2	3
4	8	12

"One third of 12 is **4**; so *two thirds* are **8**."