

Lesson 9

PERCENT OF A NUMBER

In this Lesson we will learn how to find the Amount and the Base.

In this Lesson, we will answer the following:

- 1. HOW CAN WE REPRESENT A PERCENT AS A FRACTION, OR A WHOLE NUMBER, OR A MIXED NUMBER?**
- 2. HOW CAN WE FIND THE AMOUNT WHEN WE KNOW THE BASE AND THE PERCENT?**
- 3. FRACTIONAL PERCENT**

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1. How can we represent a percent as a fraction, or a whole number, or a mixed number?

$$29\% = ?$$

We can represent a percent as a fraction whose denominator is 100.

Examples .

$$29\% = \frac{29}{100} .$$

$$60\% = \frac{60}{100} = \frac{6}{10} = \frac{3}{5} .$$

$$200\% = \frac{200}{100} = 2 .$$

$$250\% = 200\% + 50\% = 2\frac{1}{2} .$$

$$\& 225\% = 2\frac{1}{4} .$$

$$233\frac{1}{3}\% = 2\frac{1}{3} .$$

In addition, the student should know

$$12.5\% = \frac{1}{8} \quad \left(\frac{1}{8} \text{ is half of } \frac{1}{4}, \text{ which is } 25\%. \right)$$

$$37.5\% = \frac{3}{8}, \quad 62.5\% = \frac{5}{8}, \quad 87.5\% = \frac{7}{8}.$$

A percent is a ratio. Nevertheless, for purposes of calculation we can represent a percent as a fraction (or a decimal), because a fraction in turn indicates the ratio of the numerator to the denominator. The meaning of multiplying by a fraction depends upon that ratio.

2. How can we find the Amount when we know the Base and the Percent?

$$\text{Amount} = \text{Base} \times \text{Percent}$$

Example 1. How much is 75% of 104?

Answer. We could write 75% as the decimal .75. Therefore we could multiply $.75 \times 104$.

However, 75% is three quarters. While we could write

$$\frac{3}{4} \times 104$$

and calculate that, we can easily decompose 104 into $100 + 4$ - and so it is easy take three quarters of it.

$$\text{One quarter of } (100 + 4) = 25 + 1 = 26.$$

Therefore,

$$\text{Three quarters} = 3 \times 26 = 60 + 18 = 78.$$

75% of 104 is 78.

Example 2. How much is 30% of \$48?

Answer. The student should anticipate that finding 30% of a number will involve multiplication by 3 -- because 10% is \$4.80. 20% will be 2 times that. 30% will be 3 times. And so on.

$$\text{Now, } 3 \times 48 = 120 + 24 = 144.$$

Where does the decimal point go?

Since 10% is \$4.80, then 30% -- 3 times that -- is \$14.40.

Example 3. How much is 80% of \$124?

Answer. $8 \times 124 = 800 + 160 + 32 = 992.$

Where does the decimal point go?

Since 10% is \$12.40, then 80% is 9 times that: \$99.20.

Example 4. How much is 80% of \$45?

Answer. In this case, since 45 has an exact fifth part, reason that 80% is four fifths.

One fifth of 45 is 9. Therefore four fifths are four 9's, which is 36.

80% of \$45 is \$36.

Example 5. How much is 250% of 32?

Answer. $250\% = 2\frac{1}{2}.$

$$2\frac{1}{2} \times 32 = 2 \times 32 + \frac{1}{2} \times 32 \text{ -- "Two times 32 + Half of 32"}$$

$$2\frac{1}{2} \times 32 = 64 + 16$$

$$2\frac{1}{2} \times 32 = 80.$$

Example 6. How much is $37\frac{1}{2}\%$ of \$40?

Answer. $37\frac{1}{2}\% = \frac{3}{8}$. One eighth of \$40 is \$5. Therefore, three eighths are $3 \times \$5 = \15 .

Example 7. How much is 18.9% of \$314?

Answer. Use your calculator!

Example 8. Thirds. How much is $33\frac{1}{3}\%$ of 720? How much is $66\frac{2}{3}\%$?

Answer. $33\frac{1}{3}\%$ means a third. To take a third

of 720, we can decompose it as follows:

$$720 = 600 + 120.$$

A third of 600 is 200.

A third of 120 is 40.

Therefore a third of 720 is 240.

As for $66\frac{2}{3}\%$, it means two thirds. One third of 720 is 240. Therefore,

two thirds are $2 \times 240 = 480$.

Example 9. Calculator problem. How much is $66\frac{2}{3}\%$ of \$76.27?

Solution. To find *two thirds*, we must first find *one third*, and then multiply by 2. Press

7	6	.	2	7	÷	3	×	2	=
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See

50.646666666

This is approximately \$50.65.

The standard textbook method for finding a percent of a number, has been to change the percent to a decimal, and multiply. Thus, to find 24% of \$412, we are taught to change 24% to the decimal .24, and multiply times 412.

But is anyone with a calculator going to do that these days? And aren't there more important things to learn about percent? Like how much is 25% of \$412 -- without writing anything! Take half of half.

3. Fractional percent

2. What does $\frac{1}{2}\%$ mean?

Half of 1%. Half of the hundredth part.

Example 10. Distinguish the following:

- a) Half of \$600 b) $\frac{1}{2}\%$ of \$600

Answers.

a) Half of \$600 is \$300.

b) $\frac{1}{2}\%$ of \$600 means Half of 1% of \$600.

1% of \$600 is \$6.00.

Therefore, $\frac{1}{2}\%$ of \$600 is \$3.00.

Example 11. How much is $\frac{1}{2}\%$ of \$824?

Answer. \$4.12.

Again, $\frac{1}{2}\%$ means Half of 1%.

1% of \$824 is \$8.24.

$\frac{1}{2}\%$ of \$824 is \$4.12.

Notice that Half of 1% is the same as 1% of Half (1% of \$412). It does not matter which operation we do first.

Note, also, that $\frac{1}{2}\%$ of \$824 and half of \$824 have the same digits: 4, 1, 2.

Example 12. How much is $\frac{1}{4}\%$ of \$800?

Answer. $\frac{1}{4}\%$ means a quarter of 1%.

1% of \$800 is \$8.00 . $\frac{1}{4}\%$ of \$800 is \$2.00.

Example 13. Jacqueline deposited \$2000 in the bank, where the interest is $3\frac{1}{2}\%$ annually. In one year, how much interest will she earn?

Answer. We have to calculate $3\frac{1}{2}\%$ of \$2000. We can get it from 1%:

1% of \$2000 is \$20.00.

Therefore, $3\frac{1}{2}\%$ is

$$3\frac{1}{2} \times 20 = 60 + 10 = 70.$$

"Three times 20 + Half of 20."

She will earn \$70 in interest.

Example 14. Calculator problem. How much is $6\frac{1}{2}\%$ of \$104.16?

Solution. The Amount is missing. Press

1	0	4	.	1	6	×	6	.	5	%
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See:

6.7704

This is approximately, \$6.77.

If your calculator does not have a percent key, then change 6.5% to a decimal, and press = . Press

1	0	4	.	1	6	×	.	0	6	5	=
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