Lesson 3
MULTIPLY WHOLE NUMBERS
MULTIPLY DECIMALS

In this Lesson, we will answer the following:

1. **HOW DO WE MULTIPLY BY A SINGLE DIGIT?**
2. **IN WRITING, HOW DO WE MULTIPLY WHOLE NUMBERS?**
3. **HOW DO WE MULTIPLY DECIMALS?**
4. **HOW CAN WE FIND THE AREA OF A RECTANGLE?**

Let us face facts. The student will quickly replace written multiplication with a calculator. (What if you don't have a calculator? What if you don't have a pencil!) Therefore, what should the student know about written multiplication that deserves to be called educational? The student should know that it is based on the distributive property. The student should also know the basics of placing the decimal point.

1. How do we multiply by a single digit?

$$\begin{array}{r} {\scriptstyle 1\,5} \\ 628 \\ \times\ 7 \\ \hline 4396 \end{array}$$

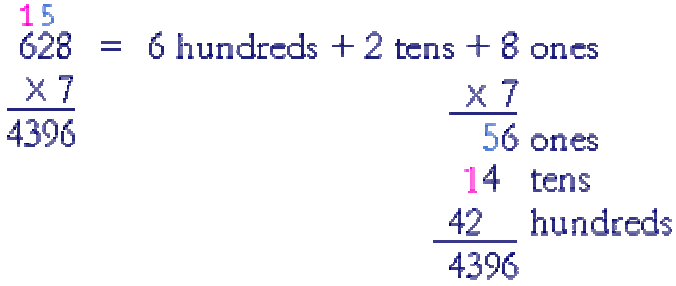
Align the multiplier (on the bottom) with the ones digit of the multiplicand, and draw a line. Then multiply each digit of the multiplicand. Write the ones digit of each product below the line. If there is a tens digit, carry it -- add it -- to the next product.

"7 times 8 is 56." Write 6, carry 5.

"7 times 2 is 14, plus 5 is 19." Write 9, carry 1.

"7 times 6 is 42, plus 1 is 43." Write 43.

We can analyze this as follows:



7 has been *distributed* to each unit of 628: to

6 hundreds + 2 tens + 8 ones.

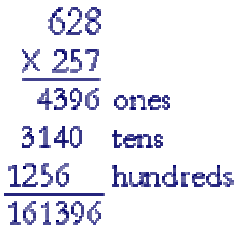
On the left (but compare the right):

7 × 8 ones = 56 ones, or simply 56. Write 6 below the line and carry the 5 onto the tens column, because the 5 is 5 tens.

Next: 7 × 2 tens = 14 tens, plus 5 are 19 tens. Write 9 and carry the 1 onto the hundreds column -- because 19 tens = 190. The carried 1 is 1 hundred.

Finally: 7 × 6 hundreds = 42 hundreds, plus 1 is 43 hundreds. Write 43.

When the multiplier has more than one digit --



-- follow the same procedure for each digit. However, when we multiply by 5 tens, the product is 3140 tens. Therefore we write 0 in the tens column.

When we multiply by 2 hundreds, the product is 1256 hundreds, and so we write 6 in the hundreds column.

2. In writing, how do we multiply whole numbers?

```
  628
X 257
 4396
3140
1256
161396
```

Write the multiplier under the multiplicand and draw a line. Multiply the multiplicand by each digit of the multiplier. Place the ones digit of each partial product in the same column as the multiplying digit. Then add the partial products.

Anticipating the next Question, if there were decimal points --

```
  6.28
X25.7
 4396
3140
1256
161.396
```

-- the multiplication would proceed in exactly the same way. In the answer, we would then separate as many decimal places as there are in the two numbers together; in this case, three.

Example 1. 0's within the multiplier.

```
   907
 × 308
  7256
272156
279356
```

8 × 907 = 7200 + 56 = 7256
0 × 907 = 0
3 × 907 = 2700 + 21 = 272150

On distributing 8 *ones*, write 6 in the *ones* column.

Any number times 0 is 0, therefore it is not necessary to write any digit in the tens column.

On distributing 3 *hundreds*, write 1 in the *hundreds* column.

It is not necessary to write rows of 0's. They add nothing to the product.

3. How do we multiply decimals?

$.2 \times 6.03$

Ignore the decimal points -- do not align them -- and multiply the numbers as whole numbers. Then, starting from the right of the product, separate as many decimal places as there are in the two numbers together.

Example 2. $.2 \times 6.03$

Solution. Ignore the decimal points. Simply multiply

$2 \times 603 = 1206$

Now we must put back the decimal points. Together, $.2$ and 6.03 have *three* decimal places. Therefore, starting from the right, separate three places:

1.206

When we ignore a decimal point, we have in effect *moved* the point to the right:

$6.03 \rightarrow 603$

We have multiplied by a power of 10.
Therefore, to compensate and name the right answer, we must divide by that power, we must separate the same number of decimal places.

Example 3. $.03 \times .002$

Solution. Ignore the decimal points.

$3 \times 2 = 6$

Together, $.03$ and $.002$ have five decimal places. Therefore, separate five places:

$.00006$

Example 4. 200 × **.**012

Solution. Ignore the decimal point. Multiply

200 × 12 = 2400

Again, to multiply whole numbers that end in 0's, first ignore the 0's, then replace them. But replace only the 0's on the end of *whole* numbers. Do not replace the 0 of **.**012

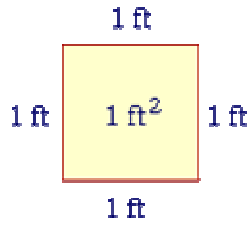
Now separate three decimal places (**.**012):

2**.**400 = 2**.**4

These are simple problems that do not require a calculator

4. How can we find the area of a rectangle?

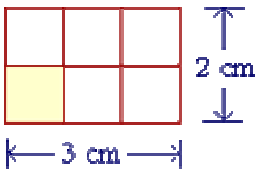
What is "1 square foot"?



1 square foot is a square figure in which each side is 1 foot.

We abbreviate "1 square foot" as 1 ft².

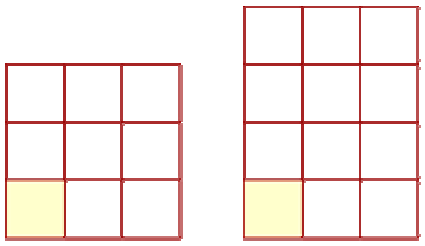
Now here is a rectangle whose base is 3 cm and whose height is 2 cm.



What do we call the small shaded square?

Since each side is 1 cm, we call it "1 square centimeter." And we can see that the entire figure is made up 2 × 3 or 6 of them!

In other words, the area of that rectangle -- the space enclosed by the boundary -- is 6 square centimeters: 6 cm².

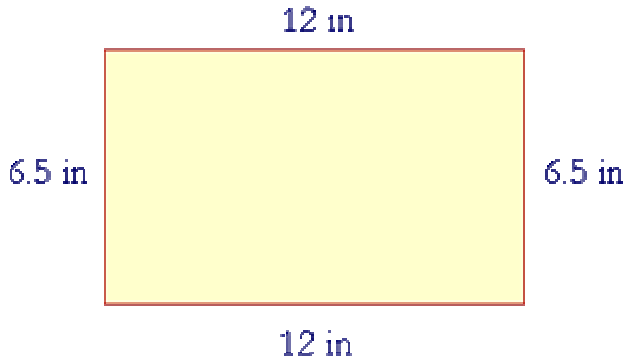


If the rectangle were 3 by 3 -- that is, if it were a square -- then it would be made up of 9 cm². If it were 3 by 4, the area would be 12 cm². And so on. In every case, to calculate the area of a rectangle, we multiply the base times the height.

Area = *Base* × *Height*

When the length is measured in centimeters, the area is measured in square centimeters: cm². And similarly for any unit of length.

We have illustrated this with whole numbers, but it will be true for any numbers.



If the base is 12 in, and the height is 6.5 in, then to find the area, multiply

12 × 6.5.

Now,

12 × 65 = 650 + 130 = 780

Therefore on separating one decimal place (6.5):

Area = 78 in²